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A NEW FORMULA FOR THE PRESSURE RECOVERY IN AN ABRUPT DIFFUSOR

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Abstract—A new formula for the pressure recovery in an abrupt expansion based on the superficial velocities of the two phases is proposed. Its predictive accuracy and that of other models from the literature are compared with new experimental data from stationary two-phase flow experiments in a diffusor with a very steep opening contour. Only the new model exhibits good agreement between predicted and measured experimental steam–water and air–water data. Furthermore, the new formula is verified by experimental data of other authors obtained with different expansion geometries and flow conditions.

Key Words: abrupt diffusor, pressure recovery, calculation, two-phase flow, two-component flow, experimental results

1. PROBLEM AND AIM

The calculation of the pressure change in pipe fittings is a crucial problem in two-phase flow applications. The abrupt diffusor expansion is such a fitting, where for low Mach number flow a part of the dynamic pressure head is recovered and a rise in static pressure is observed as the mixture expands and decelerates due to the change of flow area. Certainly, the pressure recovery is not a fully reversible effect. Therefore, one of the pressure recovery calculation problems will be to determine the efficiency of the conversion rate from dynamic pressure head into static pressure change. Therewith, the description of the two-phase pressure head necessitates a suitable density definition. This immediately leads to the difficulty of selecting an appropriate two-phase flow model. As the inertia effect of the gas phase is normally much smaller than that of the liquid phase the contribution of the gas phase to the pressure recovery may be negligible. Furthermore, it is questionable if the influence of the phase transition rate upon the pressure change must be taken into account in the case of one-component flow.

Basically, the effect of the pressure change in a subsonic two-phase flow in an expansion is described by a set of partial differential equations, the conservation equations of fluid dynamics. For the approximate prediction of the pressure recovery two different approaches are commonly used:

- Using the finite differences as the integration technique, the amount of pressure recovery may be calculated with sophisticated computer codes. Then, the conservation equations are formulated in an appropriate two-phase flow model and solved stepwise in a set of small control volumes. This has been shown for the drift–flux model by Wadle (1986).
- The other approach is to derive a one-dimensional analytical formula from the momentum or the mechanical energy balance, while simplifying all differential or integral terms, which cannot be defined explicitly.

In the following, the analytical approach for the prediction of the pressure recovery is focused upon. In the literature attempts have been made to provide usable formulations. A variety of theoretical approaches, flow model assumptions, simplifications etc. are introduced. Due to the extensive number of interacting physical effects, it is not surprising that up to now comprehensive theoretical and experimental studies are rare. As the pressure recovery in a diffusor is still of concern to engineers, a systematic theoretical and experimental investigation is performed to improve the knowledge about the problem and to provide additional experimental data.

Delhaye (1981) gives a detailed review of possible procedures for pressure recovery calculations. He starts with single-phase flows and then expands his derivations to two-phase flow. In the second section of this paper a short overview of the most important existing formulas is given, basically following Delhaye. The major restrictions of these models are elaborated later on. Hitherto reported experiments are restricted to systems such as freon–freon, air–water or steam–water. Different mixtures in the same test-section have never been tested. For this reason, two series of experiments at stationary conditions in a horizontal diffusor were carried out, one with steam–water and the other with air–water mixtures. The test apparatus is described in section 3 of the paper. The experimental data help to enlighten the flow phenomena of one-component and two-component two-phases in such an expansion. The observed effects are exemplified in section 4. Then the predictions of models from the literature are compared to the new experimental data. Since agreement with data is not satisfactory, the available models are analysed in more detail to reveal their major drawbacks and hence ease the way to finding a better approach. A new formula based on the superficial velocity concept is derived. The advantages of the proposed correlation are discussed.

After the analysis of the comparison of the theoretical with the experimental results, the new semi-empirical formulation developed using the new data basis is verified with the data of Velasco (1975) and Ferrell & McGee (1966). The paper ends with the interpretation of the results and a conclusion.

2. CORRELATIONS FOR PRESSURE RECOVERY CALCULATION IN AN ABRUPT EXPANSION

The derivation of analytical models for the pressure recovery from "first principles" necessitates some simplifications, regarding negligible or not adequately describable effects. These basic assumptions are presented below.

The common way of calculating the pressure recovery in an abrupt expansion analytically follows the Borda–Carnot theory. Figure 1 shows a sketch of an abrupt diffusor and gives the definitions of the main variables. Regardless of details inside the control volume, the pressure recovery between the upstream entrance area (1) and the outflow area (2), "far enough" downstream of the singularity, may be calculated from a momentum or mechanical energy balance. Using the Borda–Carnot theory the following assumptions are made for *single-phase flow*:

1. Stationary flow conditions.
2. No pressure loss due to wall friction.
3. Incompressibility of the fluid.
4. No influence of gravity.
5. Plane radial velocity profile.
6. Equal pressure on the flange area ($A_0 = A_2 - A_1$) as in the inlet area.
7. No inner friction (energy dissipation) in energy balance based models.

In the case of *two-phase flow* the following additional assumptions are made:

8. Pressure equilibrium between the phases ($p_L = p_G$).
9. Constant mean quality (frozen flow).

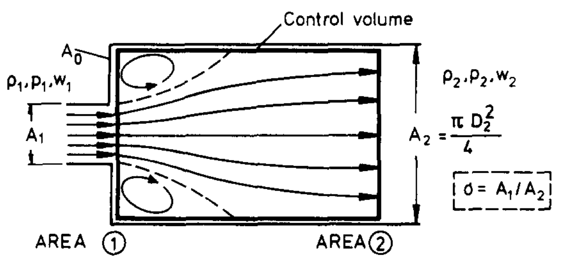

Figure 1. Ideal abrupt expansion and definitions of the main variables.

10. Constant void fraction in the control volume ($\epsilon_1 = \epsilon_2$) in the case of heterogeneous flow models (Velasco 1975; Richardson 1958).
11. Constant densities of both phases ($\rho_{L1} = \rho_{L2}$; $\rho_{G1} = \rho_{G2}$).

2.1. *Pressure Recovery in Single-phase Flow*

Starting from the momentum balance in an integral form, written as

$$\frac{\partial}{\partial t}\int \rho \mathbf{w}\, dV + \int \rho \mathbf{w}(\mathbf{w}\cdot\mathbf{n})\, dA = -\int p\mathbf{n}\, dA + \int \tau\cdot n\, dA + \int \rho \mathbf{F}\, dA, \quad [1]$$

where $\mathbf{w}$ is the velocity vector, ρ is the density, $\mathbf{n}$ is a normal vector, p is the pressure, τ is the shear tensor, $\mathbf{F}$ is the body force vector, A is the surface area and V is the integration volume; and considering assumptions 1–6, the following typical expression for the pressure recovery based on a momentum balance can be derived:

$$p_2 - p_1 = \frac{\sigma(1-\sigma)\dot{m}^2}{\rho}, \quad [2]$$

where σ is the area ratio ($\sigma = A_1/A_2$) and $\dot{m}$ is the mass velocity related to the inlet area.

On the other hand, integrating the local mechanical energy balance for the control volume gives

$$\frac{\partial}{\partial t}\int \tfrac{1}{2}\rho w^2\, dV + \int \tfrac{1}{2}\rho w^2 \mathbf{w}\cdot\mathbf{n}\, dA = -\int p\mathbf{w}\mathbf{n}\, dA + \int \nabla(\tau\cdot\mathbf{w})\cdot\mathbf{n}\, dA + \int \nabla F\rho\,\mathbf{w}\cdot\mathbf{n}\, dA - \int \tau:\nabla\mathbf{w}\, dV. \quad [3]$$

With assumptions 1–6, it follows that

$$p_2 - p_1 = \frac{(1-\sigma^2)\frac{1}{2}\dot{m}^2}{\rho}. \quad [4]$$

This is the characteristic pressure recovery formula based on the mechanical energy balance.

2.2. *Pressure Recovery in Two-phase Flow*

In two-phase flow there are models based on both homogeneous as well as on heterogeneous flow. Formulas resulting from a momentum or an energy balance are presented below.

2.2.1. *Homogeneous flow models*

The homogeneous flow assumption provides a simple calculation technique for two-phase flows. Using averaged properties, a mixture density can be defined, allowing the description of the pressure recovery with the usual equations for single-phase flow. Starting with the momentum balance yields

$$p_2 - p_1 = \sigma(1-\sigma)\dot{m}^2\left[\frac{\dot{x}}{\rho_G} + \frac{(1-\dot{x})}{\rho_L}\right] = \frac{\sigma(1-\sigma)\dot{m}^2}{\rho_h}, \quad [5]$$

where the mean density ρ_h is defined by [6] in terms of quality $\dot{x}$ and the densities of the gas and the liquid phases:

$$\frac{1}{\rho_h} = \frac{\dot{x}}{\rho_G} + \frac{1-\dot{x}}{\rho_L}. \quad [6]$$

The equation for the pressure recovery based on the mechanical energy balance reads

$$p_2 - p_1 = (1-\sigma^2)\tfrac{1}{2}\dot{m}^2\left[\frac{\dot{x}}{\rho_G} + \frac{(1-\dot{x})}{\rho_L}\right] = \frac{(1-\sigma^2)\frac{1}{2}\dot{m}^2}{\rho_h}. \quad [7]$$

Equations [5] and [7] are called the homogeneous momentum and homogeneous energy pressure recovery equations, respectively.

2.2.2. *Heterogeneous flow models*

With this physically more realistic approach, the two phases are assumed to have different mean velocities. More information about the flow parameters is, however, needed, e.g., an additional equation is necessary for the prediction of the mean void fraction from primary flow parameters. In our evaluation, the void correlation proposed by Rouhani (1969) is applied:

$$\epsilon = \left(\frac{\dot{x}}{\rho_G}\right) \Bigg/ \left[\frac{[1 + 0.12(1 - \dot{x})]}{\rho_h} + \frac{W_{rel}}{\dot{m}}\right] \quad [8]$$

with

$$W_{rel} = \frac{1.18}{\sqrt{\rho_L}} [g\sigma^*(\rho_L - \rho_G)]^{1/4}; \quad [9]$$

σ^* is the surface tension.

This formulation is recommended by Friedel (1978), who checked different void correlation predictions against a broad variety of experimental pipe flow void data. In reality, the void correlation above holds only for fully developed straight pipe flows; no procedure is available for calculating the void in the downstream area of a horizontal diffusor. Since no usable experimental void fraction data were collected, assumption 10—constant void in the diffusor—is used. It should also be noted that Velasco (1975) and Richardson (1958) obtained quite reasonable results using this assumption.

A momentum balance model based on heterogeneous flow, according to Lottes (1961), is attributed to Romie (1958). Assumptions 1–11 yield

$$p_2 - p_1 = \sigma(1 - \sigma)\dot{m}^2\left[\frac{\dot{x}^2}{\rho_G\epsilon} + \frac{(1 - \dot{x})^2}{\rho_L(1 - \epsilon)}\right] = \frac{\sigma(1 - \sigma)\dot{m}^2}{\rho_s}, \quad [10]$$

where the heterogeneous density is defined as

$$\frac{1}{\rho_s} = \frac{\dot{x}^2}{\rho_G\epsilon} + \frac{(1 - \dot{x})^2}{\rho_L(1 - \epsilon)}. \quad [11]$$

Equation [10] is analogous to [2] and [5] with the appropriate density definition for heterogeneous flows.

Lottes (1961), in his model, further simplifies the momentum balance. Using the assumption that all loss of dynamic pressure head takes place in the liquid phase he arrives at

$$p_2 - p_1 = \sigma(1 - \sigma)\dot{m}^2\left[\frac{1}{\rho_L(1 - \epsilon)^2}\right]. \quad [12]$$

Another momentum balance based on the heterogeneous model originates from Chisholm & Sutherland (1969). They recommend

$$p_2 - p_1 = \sigma(1 - \sigma)\dot{m}^2(1 - \dot{x})^2 \frac{\left(1 + \frac{C_h}{X} + \frac{1}{X^2}\right)}{\rho_L} \quad [13]$$

with

$$X = \left(\frac{1 - \dot{x}}{\dot{x}}\right)\sqrt{\frac{\rho_L}{\rho_G}} \quad [14]$$

and

$$C_h = \left[1 + \frac{1}{2}\left(\frac{\rho_L - \rho_G}{\rho_L}\right)^{1/2}\right]\left(\sqrt{\frac{\rho_L}{\rho_G}} + \sqrt{\frac{\rho_G}{\rho_L}}\right). \quad [15]$$

Besides these momentum based heterogeneous models two formulas from the mechanical energy balance are derived. The mechanical energy balance model for heterogeneous two-phase flows reads, with analogy to [4] and [7],

$$p_2 - p_1 = (1 - \sigma^2)\tfrac{1}{2}\dot{m}^2\left[\frac{\dot{x}^3}{\epsilon^2\rho_G^2} + \frac{(1 - \dot{x})^3}{(1 - \epsilon)^2\rho_L^2}\right] \cdot \left[\frac{\dot{x}}{\rho_G} + \frac{(1 - \dot{x})}{\rho_L}\right]^{-1}. \quad [16]$$

Assuming that the pressure recovery is proportional to the kinetic energies of the phases, Richardson (1958) simplifies the original formula by considering only the liquid velocity. He arrives at

$$p_2 - p_1 = (1 - \sigma^2)\tfrac{1}{2}\dot{m}^2\left[\frac{\sigma(1-\dot{x})^2}{\rho_L(1-\epsilon)}\right]. \qquad [17]$$

The predictions of the models from the literature will be compared with our experimental data. Noting that

$$2\sigma(1-\sigma) < 1 - \sigma^2 \qquad \text{if } 0 < \sigma < 1 \qquad [18]$$

it follows that the pressure recovery according to the mechanical energy balance is larger than that obtained from the momentum balance. Furthermore, the other major differences between the models will result from three sources: the definitions of the densities according to the two-phase model used; the simplifications introduced, i.e. mainly the neglect of the gas velocity; and finally, in the case of heterogeneous flow, the calculation of the void fraction from primary flow parameters. In view of these drawbacks, a new formula is derived.

2.3. Proposed Relationship for the Pressure Recovery

The pressure recovery in an abrupt expansion is caused by bulk deceleration effects. The conversion rate of dynamic pressure head into static pressure change is strongly influenced by bulk dissipation, i.e. the transfer of mechanical into thermal energy. Several investigators have shown that in two-phase flows phenomena caused by internal effects may be described in terms of the superficial velocity, e.g. in the flooding correlation of Wallis (1969) or in the flow pattern map of Mandhane *et al.* (1974). Hence, an attempt is also made to use the superficial velocity concept here as well.

The superficial velocities of the phases are defined by

$$w_{\text{sup,G}} = \frac{\dot{m}\dot{x}}{\rho_G} \qquad [19]$$

and

$$w_{\text{sup,L}} = \frac{\dot{m}(1-\dot{x})}{\rho_L}. \qquad [20]$$

The dynamic pressure head in terms of these superficial velocities is

$$p_{\text{dyn}} = \tfrac{1}{2}\rho_G\left(\frac{\dot{m}\dot{x}}{\rho_G}\right)^2 + \tfrac{1}{2}\rho_L\left[\frac{\dot{m}(1-\dot{x})}{\rho_L}\right]^2. \qquad [21]$$

Assuming the pressure recovery to be proportional to the difference in heads between areas (1) and (2) yields

$$p_2 - p_1 = K\left\{\tfrac{1}{2}\dot{m}_1^2\left[\frac{\dot{x}_1^2}{\rho_{G1}} + \frac{(1-\dot{x}_1)^2}{\rho_{L1}}\right] - \tfrac{1}{2}\dot{m}_2^2\left[\frac{\dot{x}_2^2}{\rho_{G2}} + \frac{(1-\dot{x}_2)^2}{\rho_{L2}}\right]\right\}. \qquad [22]$$

The factor K is to be experimentally adjusted. The formula may be further simplified using the mass balance and assumptions 1, 9 and 11 to give

$$p_2 - p_1 = (1-\sigma^2)\tfrac{1}{2}\dot{m}^2 K\left[\frac{\dot{x}^2}{\rho_G} + \frac{(1-\dot{x})^2}{\rho_L}\right]. \qquad [23]$$

Therewith the density definition of the new model is somewhat similar to the homogeneous density and reads

$$\frac{1}{\rho_{\text{sup}}} = \frac{\dot{x}^2}{\rho_G} + \frac{(1-\dot{x}^2)}{\rho_L}. \qquad [24]$$

This model is not derived from "first principles" (i.e. the momentum or the energy balance). It is a practical correlation with an empirical correction, which can be interpreted as an efficiency factor for the conversion of kinetic energy, formulated in terms of superficial velocities, into static

Table 1. Models for two-phase pressure recovery prediction in an abrupt expansion

1. Homogeneous Two-phase Flow Models

Momentum balance, [5]

$$p_2 - p_1 = \sigma(1-\sigma)\dot{m}^2\left[\frac{\dot{x}}{\rho_G} + \frac{(1-\dot{x})}{\rho_L}\right] = \frac{\sigma(1-\sigma)\dot{m}^2}{\rho_h}$$

Mechanical energy balance, [7]

$$p_2 - p_1 = (1-\sigma^2)\tfrac{1}{2}\dot{m}^2\left[\frac{\dot{x}}{\rho_G} + \frac{(1-\dot{x})}{\rho_L}\right] = \frac{(1-\sigma^2)\tfrac{1}{2}\dot{m}^2}{\rho_h}$$

2. Heterogeneous Two-phase Flow Models

Momentum balance

Romie model, [10]

$$p_2 - p_1 = \sigma(1-\sigma)\dot{m}^2\left[\frac{\dot{x}^2}{\rho_G\epsilon} + \frac{(1-\dot{x})^2}{(1-\epsilon)}\right] = \frac{\sigma(1-\sigma)\dot{m}^2}{\rho_s}$$

Lottes model, [12]

$$p_2 - p_1 = \sigma(1-\sigma)\dot{m}^2\left[\frac{1}{\rho_L(1-\epsilon)^2}\right]$$

Chisholm model, [13]

$$p_2 - p_1 = \sigma(1-\sigma)\dot{m}^2(1-\dot{x})^2\frac{\left(1 + \dfrac{C_h}{X} + \dfrac{1}{X^2}\right)}{\rho_L}$$

Mechanical energy balance

Mechanical energy balance, [16]

$$p_2 - p_1 = (1-\sigma^2)\tfrac{1}{2}\dot{m}^2\left[\frac{\dot{x}^3}{\epsilon^2\rho_G^2} + \frac{(1-\dot{x})^3}{(1-\epsilon)^2\rho_L^2}\right]\cdot\left[\frac{\dot{x}}{\rho_G} + \frac{(1-\dot{x})}{\rho_L}\right]^{-1}$$

Richardson model, [17]

$$p_2 - p_1 = (1-\sigma^2)\tfrac{1}{2}\dot{m}^2\left[\frac{\sigma(1-\dot{x})^2}{\rho_L(1-\epsilon)}\right]$$

3. New Formula, [23]

$$p_2 - p_1 = (1-\sigma^2)\tfrac{1}{2}\dot{m}^2 K\left[\frac{\dot{x}^2}{\rho_G} + \frac{(1-\dot{x})^2}{\rho_L}\right]$$

pressure recovery. Using experimental data of steam–water experiments, the constant turned out to be about 2/3. The implications of this value will be discussed along with the comparison of the model's prediction of air–water data and with experimental data of other authors.

One of the major advantages of the proposed correlation is that it only depends on primary flow parameters. The mass flow rate, quality and absolute pressure, from which saturation densities can be calculated, are indeed usually available. In addition, the effects of both phases are considered. Therefore, the model should cover the whole void fraction region and should not be restricted to low-void flows, where neglecting the influence of the gas phase may be justified. For better understanding, all formulas are summarized in table 1.

3. EXPERIMENTAL SETUP AND MEASUREMENT PROCEDURE

The experiments were performed in the KfK two-phase instrumentation facility, which is described in detail by John & Reimann (1979) and Kedziur (1980; Kedziur *et al.* 1980). The facility was run with steam–water as well as with air–water mixtures. The automatic loop instrumentation allows for measurement of the mass flow rates and of the entrance pressure and temperature. Therefrom, the entrance quality was derived as the third reference value. The test-section is shown in figure 2. It is manufactured entirely of stainless steel and is mounted horizontally. It consists of a tube section (ϕ 16 mm), where friction effects are dominant; an expansion section, where the pressure recovery occurs; and of a second tube section with a constant diameter (ϕ 80 mm), where separation effects should dominate. The area ratio between the inlet and the outlet tubes is $\sigma = 1:25$. The contour of the expansion pipe follows a steep hyperbolic tangent. Check

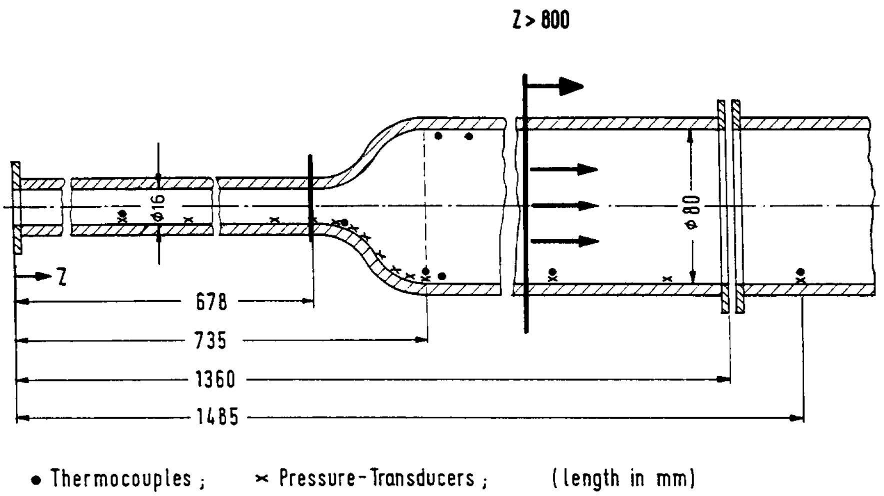

Figure 2. Test-section instrumentation location and geometry.

calculations documented in Wadle (1986) prove that such details as the control volume geometry have no effect upon the predictive quality of the pressure recovery models, originally developed for abrupt expansions.

The static pressure is measured by 16 absolute pressure transducers as shown in figure 2. These are calibrated in parallel. The maximum total error of the whole pressure recording system is < 0.05 MPa. For the evaluation of the pressure recovery, the signal from the meter position directly upstream of the expansion is used as the entrance value. The mean value of three signals in the constant diameter section downstream of the diffusor is used as the outflow value. The averaging procedure has no effect on signal quality. In the case of steam–water flow, the signals of the thermocouples, the locations of which are marked as dots in figure 2, serve as a control for the saturation condition.

4. EXPERIMENTAL RESULTS AND PHENOMENOLOGICAL INTERPRETATION

In total, including some single-phase calibration runs, 95 steam–water and 45 air–water experiments are performed. Figures 3 and 4 show the test matrices for the experiments in terms

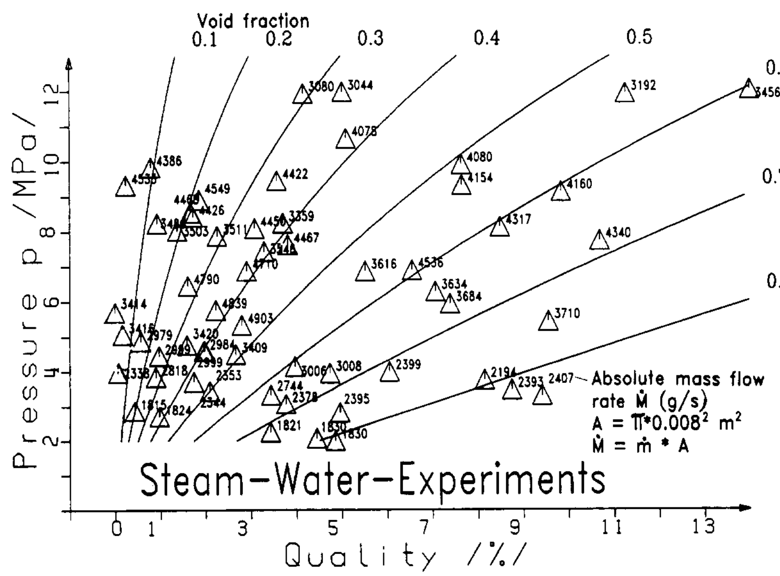

Figure 3. Test matrix for steam–water experiments.

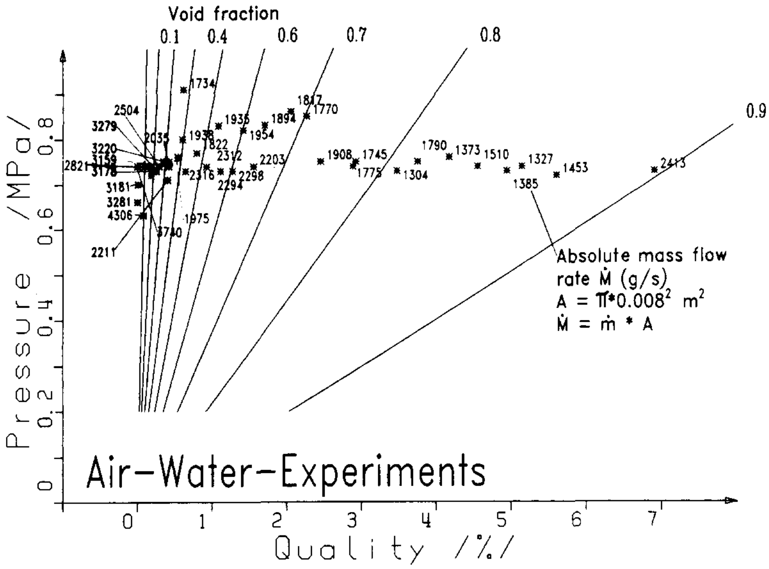

Figure 4. Test matrix for air–water experiments.

of quality and entrance pressure. The absolute mass flow rate is also indicated as well as the void fraction isolines in the diffusor inflow area—calculated assuming homogeneous flow. The whole void region is covered and the mass flow rate is increased until choked flow conditions occur. Choked flow results are excluded, however, from the evaluation of the pressure recovery models. Using air–water mixtures, the pressure variation was less.

Figure 5 shows the pressure curve of a typical run. The curve is normalized with respect to the first pressure tap in the tube. In the first part (I) the pressure decreases due to friction and the mixture is slightly accelerated. In the expansion section (II) the pressure recovery is observed, while in the downstream region the pressure values remain almost constant. As the velocities there are small, friction effects can be neglected.

There is some supplementary instrumentation in the test-section, which gives additional qualitative information. These measurements are not used for the verification of the pressure recovery models but, support the interpretation of the diffusor flow phenomena. For the signals of two γ-densitometers for void measurement with several beams, and from the radial signal profiles of movable Pitot tubes, it can be seen that the flow changes from a circular two-phase jet in region III to fully separated stratified flow in region IV. The instruments are not axially movable, therefore the transition region cannot be clearly located. The detailed experimental results are discussed by Wadle (1986).

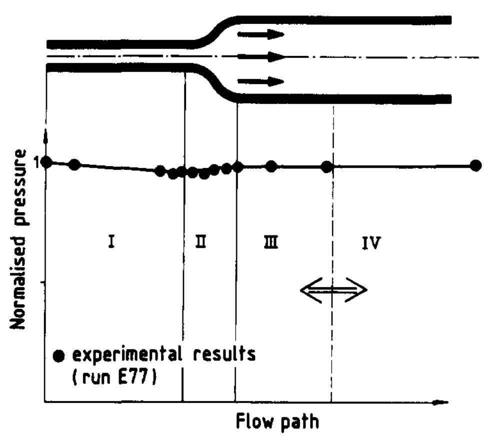

Figure 5. Normalized experimental pressure trace.

5. COMPARISON BETWEEN PRESSURE RECOVERY CALCULATIONS AND EXPERIMENTAL DATA

First, for completeness a summary of the commonly published results of single-phase flow is given. The goal of this section, however, is to compare our two-phase experimental data to various theories including the new proposal for the pressure recovery. The section concludes with the verification of our formula with the data of other authors.

5.1. Single-phase Flow

As shown in figure 6, in the case of single-phase flow, the pressure recovery normalized to the dynamic pressure head of the flow is only a function of the area ratio σ. The formulation derived from the momentum balance is physically consistent in the following sense: for $\sigma = 0$, as the area downstream increases infinitely, and for $\sigma = 1$, which means no area change, the pressure recovery vanishes in agreement with physical considerations. This consistency is not present in all energy balance based models. For $\sigma \rightarrow 0$, the nondimensional pressure drop erroneously arrives at unity, while in reality no static pressure recovery at the wall duct can be measured as the area downstream of the expansion increases to infinity. From experiments (e.g. Delhaye 1981) it is known that the formula derived from the momentum balance gives a better fit to experimental data.

5.2. Two-phase Flow

Calculated results from the seven formulas taken from the literature are compared to new data from our steam–water (figure 7) and air–water (figure 8) experiments. The abscissa is the experimental value, the ordinate gives the calculated pressure recovery. Each experiment is indicated by a symbol. The straight line ($x = y$) corresponds to ideal agreement.

Analysis of the diagrams reveals that there is a systematic deviation from the diagonal: all models derived from momentum balances, regardless of the homogeneous or heterogeneous flow assumption, give too low pressure recoveries. This also holds for the Richardson model, though it starts from mechanical energy considerations. The other models based on mechanical energy balances, however, predict values that are too high. Moreover, the data are broadly scattered across the diagram. Indeed, the models of Lottes and Chisholm & Sutherland, though giving pressure recoveries comparable to the experimental values, are too scattered. This will be discussed further with the presentation of error data, and especially the standard deviation, in table 2. Analysing the air–water experiments (figure 8), the same results and trends are found as for steam–water data.

A thorough investigation reveals that in the homogeneous models the term for the gas-phase recovery disturbs the good overall trend provided by the liquid phase. Another important result is that the heterogeneous models' main disadvantage is their dependence upon the void fraction. This contradicts experimental results, which indicates that at lower (10%) as well as at higher (90%) voids at the inlet area, the pressure recovery varies in the same region (between 0.03–0.24 MPa). In this evaluation the void fraction is calculated from loop reference data using

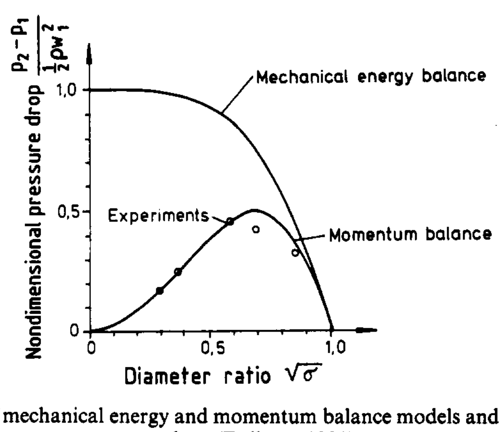

Figure 6. Predictions of mechanical energy and momentum balance models and single-phase, experimental data (Delhaye 1981).

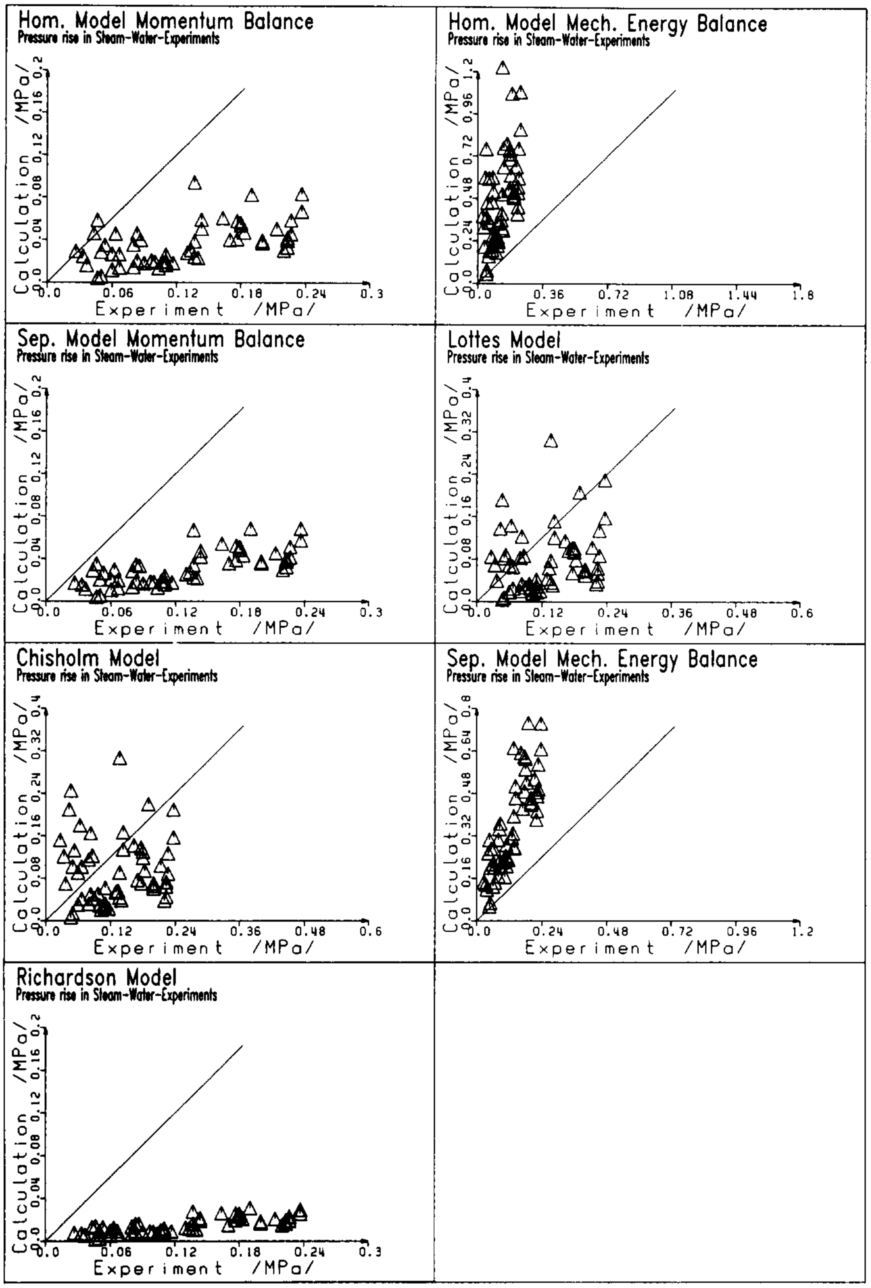

Figure 7. Predictive accuracy of the literature models in steam–water flow.

the Rouhani correlation [8]. The void data predicted from this model are always smaller than the respective void fractions calculated from the homogeneous model assumption. Therefore, in the heterogeneous flow models, slip values are used in the range of $1.14 < S < 1.97$ (steam–water mixtures) and $1.13 < S < 2.11$ (air–water mixtures). But additional checks indicate that the slip assumption (i.e. the void correlation) has a minor influence upon the calculated pressure recovery.

5.3. Results of the Comparison of the New Relationship with Our Experimental Data

In figures 9 and 10 the predictions of the proposed model are compared with our own experimental data. The correlation constant is evaluated by a least-squares fit. Almost no offset from the straight line is found and the value of K is 2/3. No dependence of this factor upon quality is observed.

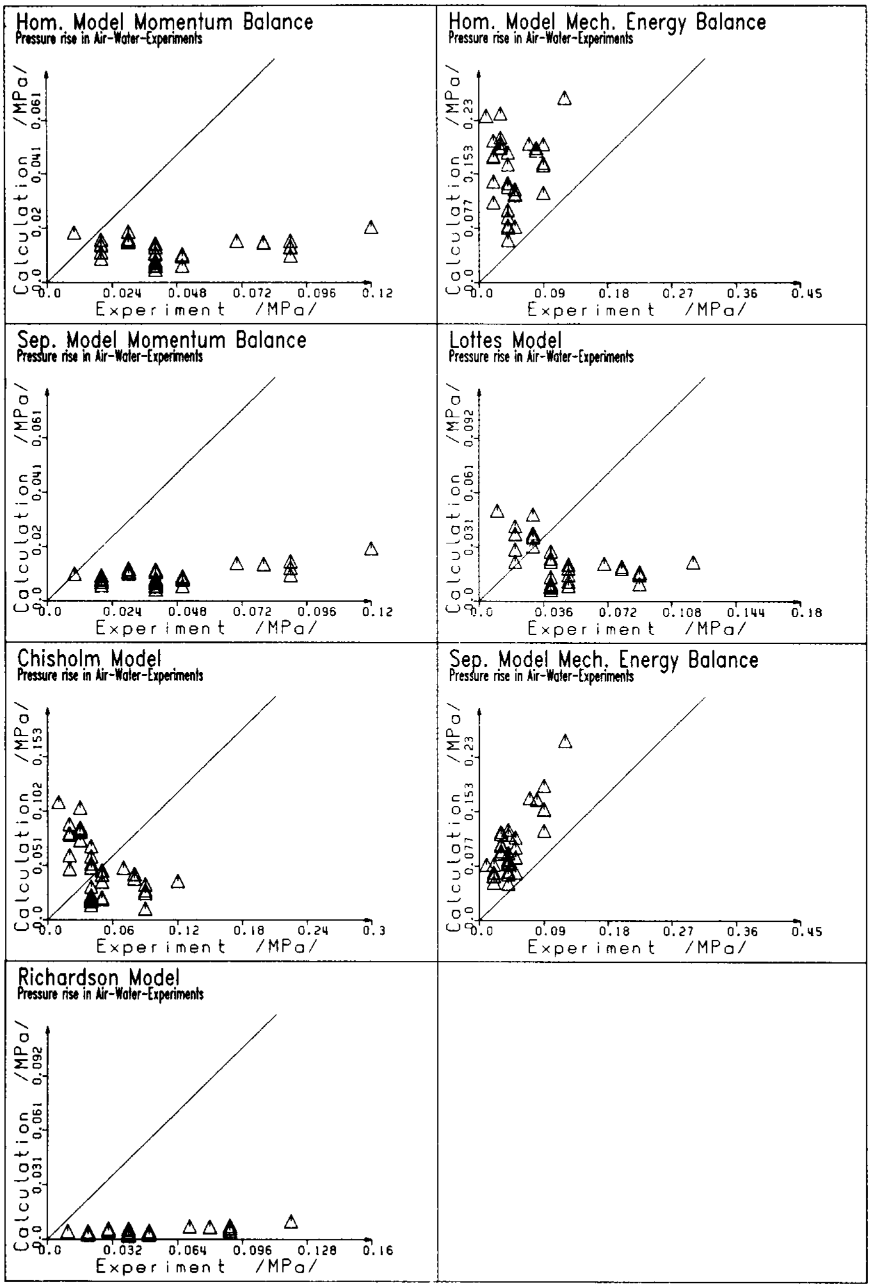

Figure 8. Predictive accuracy of the literature models in air–water flow.

Table 2. Characteristic error data of the models

	Model							
	Homog. moment A	Homog. energy B	Separate moment C	Lottes (1961) D	Chisholm & Sutherland (1969) E	Separate energy F	Richardson (1958) G	Present work
Steam–water experiments								
μ_r(%)	−67.5	322.0	−73.6	−26.6	2.1	188.9	−87.4	8.5
μ_m(%)	−90.7	1326.4	−90.9	220.8	469.2	487.8	−95.2	73.9
S_R(%)	21.7	282.1	12.9	72.2	112.6	102.9	5.6	22.2
Air–water experiments								
μ_r(%)	−67.8	303.0	−76.2	−33.1	46.4	147.3	−88.0	−4.7
μ_m(%)	−88.7	906.2	−89.2	112.5	351.0	316.2	−94.4	40.7
S_R(%)	20.2	252.6	11.0	60.0	130.7	81.1	5.2	22.0

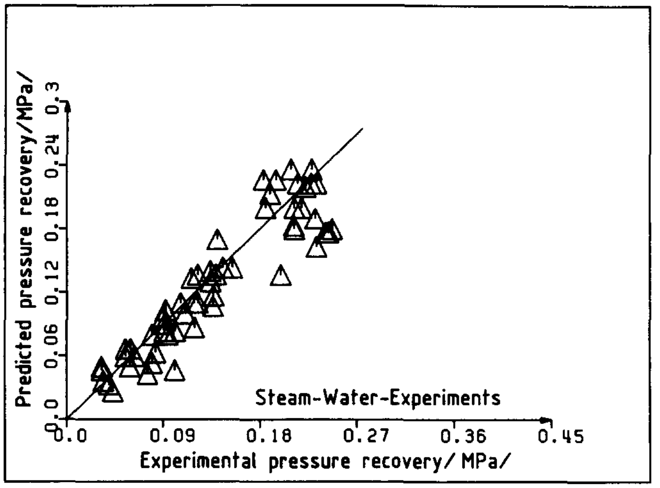

Figure 9. Predictive accuracy of the new model in steam–water flow.

The air–water data are also compared to the predictions of the correlation with the same K constant. Indeed, the switch from two-phase one-component to two-component flows does not affect the good performance of the new relationship. Hence, the constant is also valid for this system.

In table 2 the results of the error analysis of the models are included. The steam–water and air–water experiments are examined. Three characteristic error numbers are calculated for each mixture. The relative sample mean

$$\mu_r = \frac{1}{n}\left[\sum_{i-1}^{n}\left(\frac{p_{cal}-p_{exp}}{p_{exp}}\right)\right] \qquad [25]$$

gives a first guess of the model's overall centring. The formula's performance is additionally described on the basis of the maximum relative error

$$\mu_m = \max\left(\frac{p_{cal}-p_{exp}}{p_{exp}}\right) \qquad [26]$$

and the relative standard deviation

$$S_R = \sqrt{\frac{1}{n-1}\left[\sum_{i-1}^{n}\left(\frac{p_{cal}-p_{exp}}{p_{exp}}-\mu_r\right)^2\right]} \qquad [27]$$

to test whether the good sample mean value only results from compensation effects.

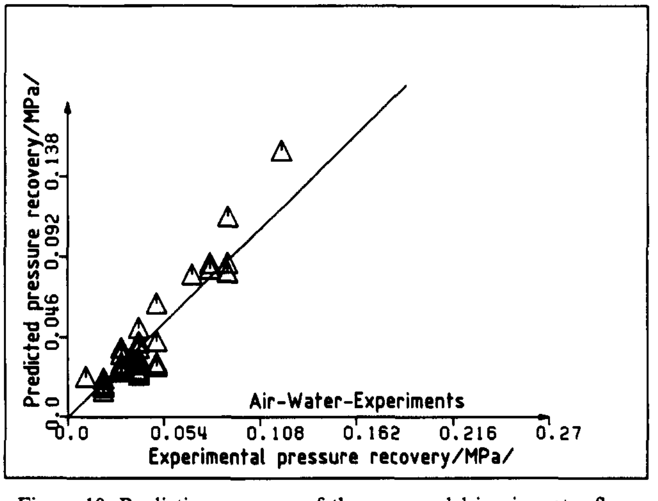

Figure 10. Predictive accuracy of the new model in air–water flow.

Table 3. Original flow parameters and experimental setup

Experiment:	Present work	Present work	Velasco (1975)	Ferrell & McGee (1966)
Fluid system:	Steam–water	Air–water	Air–water	Steam–water
Mass flux (kg/m²s)	9000–24000	4500–12000	900–4400	327–2470
Quality (%)	0–20	0–7	0–0.3	0–32
Pressure (MPa)	2–12	0.7–0.9	0.12–0.13	0.41–1.65
Flow direction	Horizontal	Horizontal	Vertical	Vertical
Data points	55	35	12	66
Area ratio	$16^2:80^2$	$16^2:80^2$	$19^2:34^2$	0.332 0.546 0.608
Expansion mode	Steep diffusor expansion		Abrupt expansion	Abrupt expansion

The comparison of our data to the predictions shows that due to the rounding of the empirical constant, the steam–water data are slightly overpredicted, whereas in the case of air–water mixtures the prediction is, on average, lower. The sample mean obtained with the Chisholm & Sutherland (1969) model is comparable to that of our model. However, the check with the maximum error data and the standard deviation shows that this is only due to the vice versa compensation of broad deviations. In comparison with common two-phase error data, mean errors of 8.5% (steam–water) and −4.7% (air–water mixtures) in the proposed new correlation seem reasonable.

5.4. Results of the Comparison of the New Relationship with the Experimental Data of Other Authors

The new model is also verified with the data of two other authors: Velasco (1975), who presented air–water experiments "at low qualities" in a vertical setup; and Ferrell & McGee (1966), who performed steam–water experiments at lower mass flow rates in a vertical test-section. Both used experimental equipment quite different from ours. In this way, it can be examined whether our model would also hold for severely altered experimental conditions. Table 3 gives an overview of the main different features and the exact parameter regions, in which the new formula is tested. As an appropriate calculation procedure, Velasco recommends the Chisholm & Sutherland (1969) model for specific experiments, while Romie's (1958) model gives the best overall prediction accuracy. Ferrell & McGee use Romie's equation for calculation purposes.

The publications of both authors include tables of experimental data for the pressure recovery. So the values used for comparison with the predictions are directly comparable to our data and no other influence (e.g. geodetical pressure drop in vertical flow) must be taken into account.

The new theory predicts the experiments of Velasco quite well, as shown in figure 11. The agreement with Ferrell & McGee's data (figure 12), on the other hand, is not as good. For the best fit the constant would have to be altered to $K = 0.83$. Even without changing the constant to 0.83

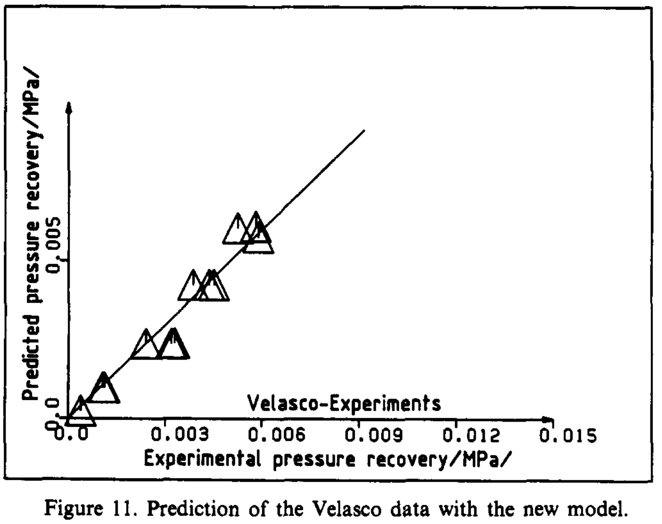

Figure 11. Prediction of the Velasco data with the new model.

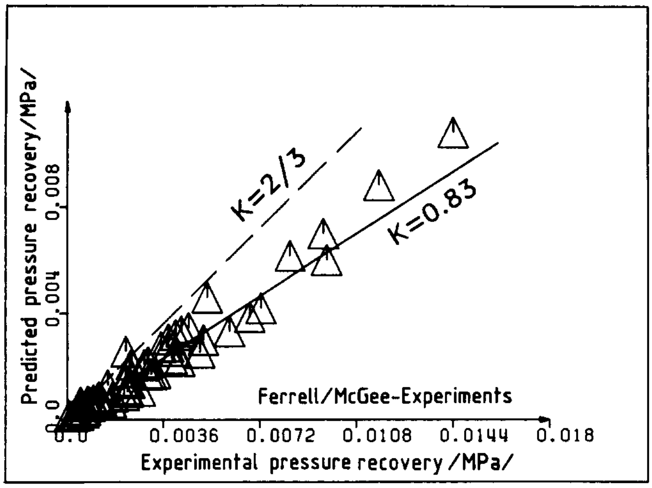

Figure 12. Prediction of the Ferrell & McGee data with the new model.

the new correlation gives better results than Romie's equation, which is recommended by Ferrell & McGee. These authors report an error number for this prediction in the range of $\pm 40\%$ without further illustrating the calculation procedure.

6. DISCUSSION OF THE RESULTS OF THE COMPARISON AND FINAL CONCLUSIONS

A theoretical and experimental study on the pressure recovery in an abrupt expansion is presented. During the experiments, the inlet flow conditions and the fluids were varied. Common prediction models for pressure recovery calculation included in the literature are checked. A new formula is derived for the pressure recovery. Our model does not require void data, it depends only upon primary flow parameters, and a second auxiliary correlation for the void fraction is not necessary. The formula introduces a new two-phase density definition. No simplification concerning the gas phase is needed. Only this new model, which assumes that the pressure recovery is proportional to the dynamic pressure head defined in terms of superficial velocities, can appropriately describe both our steam–water and air–water experiments. Finally, our new model is verified by the experiments of other authors and gives better predictions than those of previous correlations.

Some conclusions derived from the results presented are as follows:

1. The explanation that pressure recovery models based on the momentum equation predict values far less than the experimental values, while mechanical energy equation based models yield pressure recoveries far above the experimental values, may result from the different forms of the friction terms in both conservation equations. The momentum balance describes only forces acting on the fluid system at the control volume boundaries. Deriving the pressure recovery formulas these friction effects at the duct walls are neglected. Effects inside the control volume are not taken into account explicitly. Nevertheless, bulk effects are implicitly maintained. On the other hand, starting from the mechanical energy balance, the dissipative effect must be treated explicitly. As mentioned above, no appropriate model exists for this term and, therefore, it is neglected in the models tested in the present study. However, from experimental data we see that there is a significant, but not a total, dissipative loss due to strong turbulence in the control volume. The result leads to the conclusion that neither the implicit inclusion of dissipation (momentum balance) nor the explicit neglect (energy balance) are correct.
2. In the light of the explanations concerning dissipation, the constant factor of our model may be interpreted as an efficiency factor, which describes the rate at which the kinetic energy of the two-phase flow, formulated in terms of the superficial velocities, is reversibly converted into a pressure recovery in an abrupt expansion, compared to the ideal dissipation-free recovery. The result that the constant of the proposed correlation does not depend upon quality indicates that

the chosen density definition is appropriate for describing two-phase pressure recovery experiments in a diffusor.

3. Another important feature can be derived, especially from results of the models of Lottes and Richardson. From these it may be concluded that neglecting the gas phase in one way or another is not a valid assumption for pressure recovery calculations. In this case, the results are too widely scattered or far too low. This idea is supported by the results of a more sophisticated examination, where a remarkable influence of the quality upon the pressure recovery is found. Furthermore, the basic idea that the influence of both phases must be included in a formula which attempts to cover the whole void region is justified.
4. As the predictive accuracy of the proposed relationship applies to steam–water as well as to air–water experiments, the idea arises that phase-transition effects could be of minor influence. This assumption is supported by the fact that the change of flow components does not affect the predictive accuracy of our model, and by our additional calculations with a computer code system (Wadle 1986). There, we compared pressure recovery data calculated with a finite difference code with our experimental data. As the basic modelling assumptions we used the homogeneous flow model and different drift–flux models (heterogeneous flow). We found that the homogeneous model overpredicts the recovery by more than one order of magnitude, while a drift–flux model gives good results. This finally leads to the conclusion that the pressure recovery in the diffusor is essentially a mechanical effect.
5. From the comparison of the proposed correlation with the experiments of other authors, it can be concluded that the new model, which is based on the use of superficial velocities, describes air–water experiments and steam–water data at relatively high mass flow rates. The main difference between Ferrell & McGee's (1966) data and our own is the lower mass flow rate. Lower flow rates will result in lower velocities and less turbulence. It emerges from the test of the literature models that a major problem is to include appropriate dissipation effects in the formula. This may be less important at lower velocities. With a slight change in the constant, however, experiments with lower mass flow rates can be satisfactorily predicted.
6. From the test of our correlation with two different experimental data sets of our own, and the data of two other authors with quite different test apparatuses, and the cross check as far as the mixture components are concerned, it may be speculated that the formula may hold for a variety of mixtures.

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